Rozwiązania o zmiennych rozdzielonych (Ćwiczenia ~~25.05.2013~~ 8.06.2013)

1. $y' = y-1$; $\frac{dy}{dx} = y-1 \;/:(y-1)\cdot dx \rightarrow \frac{dy}{y-1} = dx \Rightarrow \int \frac{dy}{y-1} = \int dx \rightarrow$
$\ln|y-1| = x+C \rightarrow y-1 = e^{x+C} \rightarrow \underline{y = e^{x+C}+1}$

2. $y' = \frac{1-x}{1+y}$; $\frac{dy}{dx} = \frac{1+x}{1+y} \;/\cdot(1+y)\cdot dx \rightarrow (1+y)dy = (1+x)dx \rightarrow$
$\ln|1+y| = \ln|1+x| + \ln C \rightarrow \ln|1+y| = \ln C|1+x| \rightarrow 1+y = C|1+x|$
$y = C|1+x| - 1$

3. $y' = e^{x+y}$; $\frac{dy}{dx} = e^x \cdot e^y \;/:e^y\cdot dx \rightarrow \frac{dy}{e^y} = e^x dx \rightarrow \int \frac{dy}{e^y} = \int e^x dx$
$-e^{-y} = e^x + C \rightarrow \frac{1}{e^y} = -e^x + C_1 \rightarrow e^y = \frac{1}{C_1 - e^x} \rightarrow \underline{y = \ln\left(\frac{1}{C_1 - e^x}\right)}$

4. $y' = \frac{2x}{e^y}$; $\frac{dy}{dx} = \frac{2x}{e^y} \rightarrow e^y dy = 2x dx \rightarrow \int e^y dy = \int 2x dx \rightarrow e^y = x^2 + C$
$y \ln e = \ln(x^2+C) \rightarrow \underline{y = \ln(x^2+C)}$

5. $y' = \frac{x}{y}$; $y(1)=2$; $\frac{dy}{dx} = \frac{x}{y} \rightarrow y dy = x dx \rightarrow \frac{y^2}{2} = \frac{x^2}{2} + C$
$y^2 = x^2 + C_1 \rightarrow y = \pm\sqrt{x^2+C_1}$; $2 = \pm\sqrt{1^2+C_1} \rightarrow 4 = 1 + C_1 \rightarrow \underline{C_1 = 3}$
$\underline{y = \pm\sqrt{x^2+3}}$

6. $y' = \sin 2x$; $\frac{dy}{dx} = \sin 2x \rightarrow dy = \sin 2x dx \rightarrow y = \int \sin 2x dx \rightarrow$
$y = \sin^2 x + C$ war. początk: $y(0) = \frac{\pi}{2}$ wtyc $\frac{\pi}{2} = \sin^2 0 + C$
zatem $C = \frac{\pi}{2}$. ostatecznie $\underline{y = \sin^2 x + \frac{\pi}{2}}$

7. $y' = 2xy^2$; $\frac{dy}{dx} = 2xy^2 \;/:y^2\cdot dx \rightarrow \frac{dy}{y^2} = 2x dx \rightarrow -\frac{1}{y} = 2\frac{x^2}{2} + C$
$\frac{1}{y} = -x^2 + C_1 \rightarrow \underline{y = \frac{1}{C_1 - x^2}}$

8. $y' = -y$; $\frac{dy}{dx} = -y \;/:y\cdot dx \rightarrow \frac{dy}{y} = -dx \rightarrow \ln|y| = -x + C$
$y = e^{-x+C} = \frac{e^C}{e^x} = \underline{C_1 \cdot e^{-x}}$

9. $y' = -2xy$; $\frac{dy}{dx} = -2xy \;/:y\cdot dx \rightarrow \frac{dy}{y} = -2x dx \rightarrow \ln|y| = -2\frac{x^2}{2} + C$
$y = e^{x^2+C} \Rightarrow y = e^C \cdot e^{x^2} \rightarrow \underline{y = C_1 e^{x^2}}$

10. $y' = x-1$; $\frac{dy}{dx} = x-1 \;/\cdot dx \rightarrow dy = (x-1)dx \rightarrow \underline{y = \frac{x^2}{2} - x + C}$